परमाणु ऊर्जा शिक्षण संस्था

(परमाणु ऊर्जा विभाग का स्वायत्त निकाय, भारत सरकार)

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CLASS XII (MATHEMATICS)

CHAPTER – 09

DIFFERENTIAL EQUATIONS

MODULE – 1/3

DIFFERENTIAL EQUATIONS

- **BASIC CONCEPTS:-**

- **INDEPENDENT & DEPENDENT VARIABLES:**
- If y = f(x), then x is independent variable & y is dependent variable.
- Example: (i) $y = 3x^2 + 5\,x\ - 1$
- (ii) y = $Sin^2x\ - 6\,Sinx + 7$
- (iii) y = 5 e^x + 4

DIFFERENTIAL EQUATION:

- An equation involving derivative (derivatives) of the dependent variable with respect to independent variable (variables) is called a differential equation.
- Example:- (i) $\frac{\mathrm{dy}}{\mathrm{dx}} + 3\mathrm{xy} = \mathrm{Cosx}$
- (ii) $\frac{d^2y}{dx^2} + \mathrm{x}\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{e}^y$
- (iii) $\left(\frac{d^3y}{dx^3}\right)^2 + 2\mathrm{x}\frac{d^2y}{dx^2} + \mathrm{Sin}\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right) = \mathrm{x}^2 logx$

<u>ORDINARY DIFFERENTIAL EQUATION:</u>

- A differential equation involving derivatives of the dependent variable with respect to <u>**only one**</u> independent variable is called an ordinary differential equation.
- Example:- (i) $y\left(\frac{dy}{dx}\right)^3 - 5\tan y = \mathrm{Cos}x$
- (ii) $\left(\frac{d^2y}{dx^2}\right)^4 + x\left(\frac{dy}{dx}\right)^5 = e^y$
- (iii) $\left(\frac{d^3y}{dx^3}\right)^{\frac{3}{2}} - 7y^3\left(\frac{dy}{dx}\right) = 3x^2$

PARTIAL DIFFERENTIAL EQUATIONS

- There are differential equations involving derivatives with respect to more than one independent variables, called partial differential equations but at this stage we shall confine ourselves to the study of ordinary differential equations only.

- We will use the term ‘differential equation’ for ‘ordinary differential equation’.

ORDER OF A DIFFERENTIAL EQUATION:

- Order of a differential equation is the order of the highest order derivative occurring in the differential equation.
- Example:- (i) $\left(\frac{d^2y}{dx^2}\right)^4 + \mathrm{x}\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^5 = \mathrm{e}^y$ Order = 2
- (ii) $\frac{d^3y}{dx^3} + \mathrm{Sin}\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right) = \mathrm{x}^2 logx$ Order = 3
- (iii) $t^2\frac{d^2s}{dt^2} - st\frac{ds}{dt} = \frac{d^4s}{dt^4}$ Order = 4
- (iv) $[(y)''']^2 + [(y)'']^3 - [(y)']^4 + y^5 = 0$ Order = 3

DEGREE OF A DIFFERENTIAL EQUATION:

- Degree of a differential equation is defined if it is a polynomial equation in its derivatives.
- Degree of a differential equation is the highest power (positive integer only) of the highest order derivative in it.
- Example:- (i) $\left(\frac{d^2y}{dx^2}\right)^4 + x\left(\frac{dy}{dx}\right)^5 = e^y$ Degree = 4
- (ii) $\frac{d^3y}{dx^3} + \text{Sin}\left(\frac{dy}{dx}\right) = x^2 logx$ Degree = not defined
- (iii) $\left[1 - \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = 3\ \frac{d^2y}{dx^2}$ Degree = 2
- (iv) $\text{Cos}\left(\frac{d^2y}{dx^2}\right) + 7\ x\ \frac{dy}{dx} = 5$ Degree = not defined

SOLUTION OF DIFFERENTIAL EQUATION:

- A relation between involved variables, which satisfy the given differential equation is called its solution.
- **GENERAL SOLUTION:**
- The solution which contains as many arbitrary constants as the order of the differential equation is called the general solution.
- Example:- y = A x^5 is the general solution of the differential equation x$\frac{dy}{dx} - 5y = 0$, because the general solution contains one arbitrary constant "A" and the order of the given differential equation is also "one".

PARTICULAR SOLUTION:

- The solution which is free from arbitrary constants is called a particular solution.
- Example:-
- $y = 9\,x^5$ is a particular solution of the differential equation $x\,\frac{dy}{dx} - 5\,y = 0$, because this solution has been obtained by giving a particular value "9" to the arbitrary constant "A" in the general solution.

ILLUSTRATIVE EXAMPLES

- 1. Verify that y = $x^2 + 2x + C$ is a solution of the differential equation $y' - 2x - 2 = 0$.
- Ans:- y = $x^2 + 2x + C$
- Diff. w.r.to x
- $\frac{dy}{dx}$ = 2x +2
- Now the differential equation is $y' - 2x - 2 = 0$ ---- (i)
- L.H.S of (i) = $\frac{dy}{dx}$ - 2x -2

  = 2x +2 - 2x -2 = 0 = R.H.S of (i)

  Therefore y = $x^2 + 2x + C$ is a solution of the differential equation $y' - 2x - 2 = 0$.

2. Verify that the function y = A Cos 2x−B Sin $2x$ is a solution of the differential equation $\frac{d^2y}{dx^2}$ + 4 y = 0.

- Ans:

y = A Cos 2x−B Sin $2x$

Diff. w.r.to x

$\frac{dy}{dx}$ = A(- Sin 2x) .2 – B (Cos 2x) . 2

Again diff. w.r.to x

$\frac{d^2y}{dx^2}$ = A(- Cos 2x) .4 – B (- Sin 2x) . 4

Now the differential equation is $\frac{d^2y}{dx^2}$ + 4 y = 0 --------(i)

L.H.S of (i) = $\frac{d^2y}{dx^2}$ + 4 y = -4 A Cos 2x + 4 B Sin 2x + 4 (A Cos 2x−B Sin $2x$)
= 0 = R.H.S of (i)

Therefore y = A Cos 2x−B Sin $2x$ is a solution of the differential equation $\frac{d^2y}{dx^2}$ + 4 y = 0.

3. Verify that the $y = c\, e^{tan^{-1}x}$ is a solution of the differential equation $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$.

- Ans:

$$y = c\, e^{tan^{-1}x}$$

Diff. w.r.to x

$$\frac{dy}{dx} = c\, e^{tan^{-1}x} \cdot \frac{1}{1+x^2}$$

$$(1+x^2)\frac{dy}{dx} = c\, e^{tan^{-1}x} \quad \text{or} \quad (1+x^2)\frac{dy}{dx} = y$$

Again diff. w.r.to x $\quad (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{dy}{dx}$

$(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$. which is same as the given differential equation.

- So, $y = c\, e^{tan^{-1}x}$ is a solution of the differential equation

$$(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0.$$